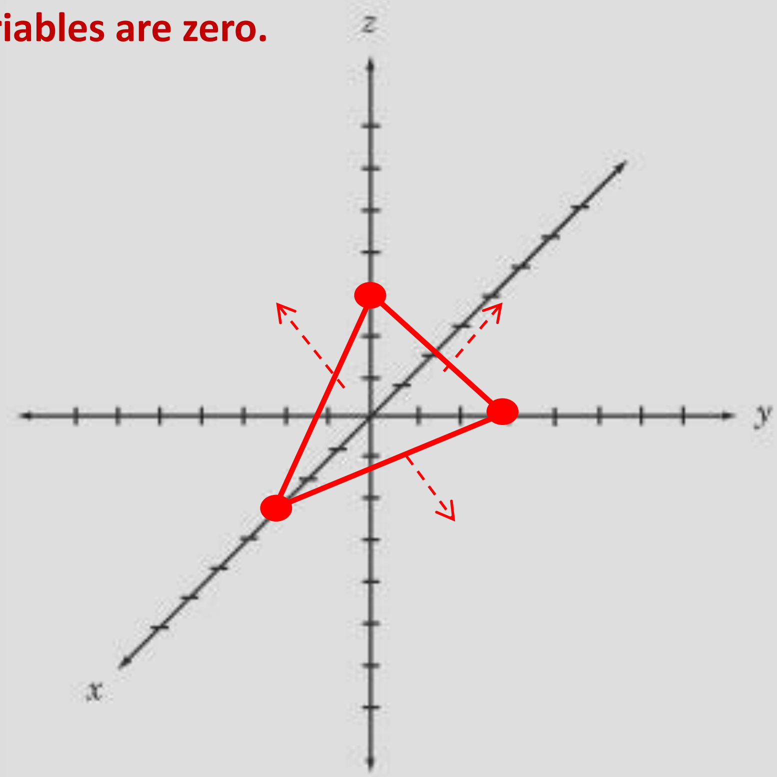
The graph of an equation in three variables, such as,
$Ax + By + Cz = D$ where A,B, and C are not all zero, is a plane.

To graph an equation with three variables, using intercepts:
- find the x, y, and z intercepts {by substituting 0 in for the other variables}
- connect the three intercepts with a triangle

When a line intercepts an axis, the value of the other variables are zero.

Graph the equation:
$x + y + z = 3$



x-intercept

$x + y + z = 3$ {the equation}
$x + 0 + 0 = 3$ {substituted 0 for y and z}
$x = 3$ {combined like terms}
coordinates are (3, 0, 0)

y-intercept

$x + y + z = 3$ {the equation}
$0 + y + 0 = 3$ {substituted 0 for x and z}
$y = 3$ {combined like terms}
coordinates are (0 , 3 , 0)

z-intercept

$x + y + z = 3$ {the equation}
$0 + 0 + z = 3$ {substituted 0 for x and y}
$z = 3$ {combined like terms}
coordinates are (0 , 0 , 3)

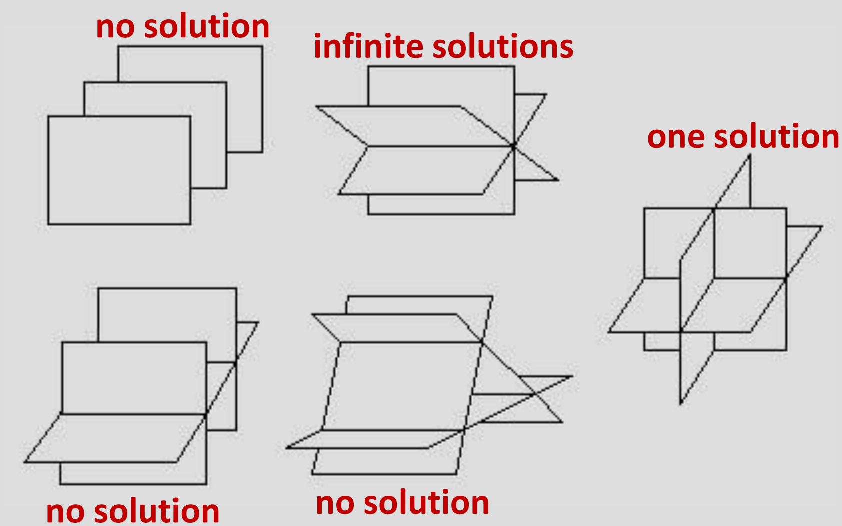
You can show the solutions of a three variable system, graphically, as the intersection of planes.

A system of three equations may have:

one solution: one point of intersection

no solution: no point of intersection {parallel planes}

infinite solutions: intersect in a line {containing an infinite number of points}



Steps to solve system of three linear equations:

1.) choose any two equations and eliminate one variable

2.) choose two different equations and eliminate the same variable

3.) use the two new equations to solve for a variable

4.) keep substituting until all variables are solved for

Example

$$-x + 3y + z = -10$$
$$3x + 2y - 2z = 3$$
$$2x - y - 4z = -7$$

1. $-x + 3y + z = -10$

$3x + 2y - 2z = 3$

$2x - y - 4z = -7$

1.) choose two equations and eliminate one variable

$3x + 2y - 2z = 3 \rightarrow 3x + 2y - 2z = 3$

$2(2x - y - 4z = -7) \rightarrow 4x - 2y - 8z = -14$

$7x - 10z = -11$

2.) choose two different equations and eliminate the same variable {y}

$-x + 3y + z = -10 \rightarrow -x + 3y + z = -10$

$3(2x - y - 4z = -7) \rightarrow 6x - 3y - 12z = -21$

$5x - 11z = -31$

3.) use the two new equations to solve for a variable

$-5(7x - 10z = -11) \rightarrow -35x + 50z = 55$

$7(5x - 11z = -31) \rightarrow 35x - 77z = -217$

$-27z = -162$

z = 6

4.) keep substituting until all variables are solved for

substitute 6 in for z, into any equation containing z and one other variable

$7x - 10z = -11$

$7x - 10(6) = -11$

$7x - 60 = -11$

$+60 \quad +60$

$7x = 49$

x = 7

substitute 6 in for z and 7 in for x into any equation with x,y, and z

$-x + 3y + z = -10$

$-7 + 3y + 6 = -10$

$3y - 1 = -10$

$3y = -9$

y = -3

2. $x + y + z = 1$

$x + 3y + 7z = 13$

$x + 2y + 3z = 4$

1.) choose two equations and eliminate one variable

$-1(x + y + z = 1)$ → $-x - y - z = -1$

$x + 3y + 7z = 13$ → $\underline{x + 3y + 7z = 13}$

$2y + 6z = 12$

2.) choose two different equations and eliminate the same variable {x}

$-1(x + 3y + 7z = 13)$ → $-x - 3y - 7z = -13$

$x + 2y + 3z = 4$ → $\underline{x + 2y + 3z = 4}$

$-y - 4z = -9$

3.) use the two new equations to solve for a variable

$2y + 6z = 12$ → $2y + 6z = 12$

$2(-y - 4z = -9)$ → $\underline{-2y - 8z = -18}$

$-2z = -6$

$z = 3$

4.) keep substituting until all variables are solved for

substitute 3 in for z, into any equation containing z and one other variable

$2y + 6z = 12$

$2y + 6(3) = 12$

$2y + 18 = 12$

$-18 \quad -18$

$2y = -6$

$y = -3$

substitute 3 in for z and -3 in for y into any equation with x,y, and z

$x + y + z = 1$

$x + (-3) + 3 = 1$

$x = 1$